



## TIME AND THE VALUE OF MONEY

Most agribusiness managers are familiar with the terms compounding, discounting, annuity, and capitalization. That is, most agribusiness managers have an intuitive understanding that each term implies some relationship between the value of money and the passage of time, but few would be able to accurately describe that relationship. For example, you probably recognize that the present value of a \$100 promissory note payable one year hence is something less than the note's face value. You also probably recognize that the present value of a series of twelve monthly revenues of \$10 to be received in the future is something less than the sum total of those revenues. However, are you able to provide a detailed explanation of the relationship between the value of money and the passage of time? Probably not-except to state your awareness of the existence of such a relationship.

Every agribusiness manager is faced with making decisions concerning alternative uses of available capital. A farm supply business manager may have to decide whether to enlarge his current product line or enlarge his facilities. A fruit warehouse manager may have to decide whether to invest funds in new, more efficient equipment with an immediate return or in expanded C.A. facilities which may not generate a net return for several years. Or, a grain elevator manager may have to decide whether to sell some used grain trucks now at a stated price, or wait a few more years and sell at some unknown price. Regardless, all three managers are faced with a decision, which requires an understanding

of the relationship between time and the value of money.

Exactly what is this time-value relationship and what should an agribusiness manager know about it to make better decisions? Actually, there exists no single time-value relationship. Instead there are several, including: a) future value of a present sum, b) present value of a future sum, c) present value of an annual revenue, d) present value of an annuity for a given time, and e) future value of an annuity for a given time. Every agribusiness manager should have at least a working knowledge of these five simple relationships. Through a series of brief illustrations, it is hoped that this paper will provide such knowledge.

### Future Value of a Present Sum

Let's assume that you are the manager of a fertilizer supply firm, which specializes, in aerial applications. At the present time your firm's major investment is in aircraft with a remaining useful life of about five years. In order to facilitate the purchase of new aircraft, you decide to place \$20,000 in a special bank account yielding six percent interest per annum. Your plans are to withdraw the \$20,000, plus interest in five years and use this amount to help buy the new aircraft. Allowing the interest to accumulate over the entire period, how much money will you be able to withdraw in five years? This future value of a present sum can be determined by using the following formula:

$$A = P(1+i)^n$$

where:  $A$  = future value  
 $P$  = present sum  
 $i$  = interest rate (per conversion period)  
 $n$  = number of conversion periods

In our illustration, we see that the present sum (\$20,000) is converted (period for which interest is paid) five times at an interest rate of six percent. Moreover, the formula becomes:

$$\begin{aligned} &= \$20,000 (1.06)^5 \\ &= \$20,000 (1.3382) \\ &= \$26,764 \end{aligned}$$

The factor  $(1.06)^5$  can be hand calculated, but it can be more easily obtained from a compound interest table found in the appendix of most mathematics or statistics books, e.g., according to the appropriate table values for  $i$  and  $n$ ,  $(1.06)^5$  equals 1.3382. As shown above, \$26,764 could be withdrawn from the special account at the end of the five-year period.

### Present Value of a Future Sum

Suppose a dairy cooperative purchases its supplies from a large federated supply cooperative. This year, the dairy cooperative received a certificate of equity for \$20,000, which represents the cooperative's share of the federation's earnings. At the present, the federation is paying its certificates (revolving its equities) in five years. Let's assume that you are the manager of the dairy cooperative and that your board of directors has asked you to select a representative discount rate and determine the present value of the \$20,000 certificate. Selecting a discount rate of six percent, you could use the following formula to calculate the present value:

$$P = \frac{A}{(1+i)^n}$$

where:  $P$  = present value  
 $A$  = future sum  
 $i$  = discount rate (per conversion period)  
 $n$  = number of conversion periods

In our illustration, we see that the future sum of \$20,000 is being discounted at six percent for five conversion periods and our formula becomes:

$$\begin{aligned} P &= \frac{\$20,000}{(1.06)^5} \\ &= \frac{\$20,000}{1.3382} \end{aligned}$$

The factor  $(1.06)^5$  can be taken from the compound interest table noted earlier. Often, however, it will be easier to solve the formula using a different table -- one called the discount table, showing the present values for various values of  $i$  and  $n$ , and also found in most mathematics books. It can be shown that the following mathematical relationship exists:

$$\frac{1}{(1+i)^n} = (1+i)^{-n}$$

Substituting into our formula we find that:

$$\begin{aligned} P &= A(1+i)^{-n} \\ &= \$20,000 (.7473) \\ &= \$14,946 \end{aligned}$$

The present value of the certificate is \$14,946 and can be expressed in two ways: a) you have concluded that you would be just as well off to receive \$14,949 now as to receive \$20,000 five years from now, or b) you are concluding that it would take \$14,946 invested today at six percent interest per annum to yield \$20,000 in five years.

## Interest or Discount

So far we have used the terms interest and discount. One should be able to distinguish between the two. In our first illustration, we started with a present sum and moved into the future to determine the value of that sum for a future time. In fact, when moving from an earlier time to a future time, interest is always involved. In our second illustration, the movement was reversed, i.e., we started with a future sum and determined the value of the sum at the present time. Therefore, when moving from any future time to an earlier time, discounting is always involved.

## Present Value of Constant Annual Revenue

Now let's assume that you are the manager of a fruit warehouse. Your firm is stacking its loose fruit boxes on a one-acre plot just to the rear of your warehouse. Had your firm not owned this plot of land, it would probably have had to pay an annual rent of \$200 for the privilege of stacking its boxes there. Considering the fact that ownership of this acre allows your firm to avoid this rental fee, this acre is earning your firm an equivalent of \$200 per year. If you expect all firm assets to create a rate of return of at least ten percent, what is the capitalized (present) value of the one-acre storage area? To answer this question, one should use the following formula:

$$P = \frac{R}{i}$$

where  $P$  = present (or capitalized) value  
 $R$  = revenue (per conversion period)  
 $i$  = discount rate (per conversion period)

In our illustration, we see that:

$$P = \frac{\$200}{.10} = \$2,000$$

In other words, according to your stated minimum rate of return (discount), the present value of your storage area is \$2,000.

Moreover, this can be described in two ways:

a) it is the present value of an annual revenue of \$200 for an indefinite time, or b) it is the sum you would have to invest at ten percent interest in order to obtain an annual return of \$200 for an indefinite number of years.

## Present Value of an Annuity for a Given Time

Let's assume that as the manager of a trucking firm you are considering the purchase of a new two-ton truck. According to your records, you can expect the new truck to produce annual net revenue of \$1000 for its useful life of ten years, after which no salvage value exists. As in our earlier example, you have established a minimum requirement of a ten percent rate of return before you will make any new investments. You now wish to determine the maximum amount you could pay for the new truck and still achieve your expected rate of return.

To solve this problem, one should use the following formula:

$$P = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$

where:  $P$  = present value  
 $R$  = revenue (per conversion period);  
 $i$  = interest rate (per conversion period)  
 $n$  = number of conversion periods

In our illustration, the formula becomes:

$$\begin{aligned} P &= \$1,000 \left[ \frac{1 - (1.10)^{-10}}{.10} \right] \\ &= 1,000 \left( \frac{1 - .3855}{.10} \right) \\ &= \$6,145 \end{aligned}$$

Based on your expectations and your required rate of return, you could pay up to \$6,145 for the new truck and still have a satisfactory investment. Actually, this answer could be interpreted two ways: a) it is the present value of ten annual revenues of \$1,000 each, with revenues received at the end of each year and with a ten percent discount rate, and b) it also represents the dollar amount, which if invested at ten percent interest, will be completely exhausted after withdrawals of \$1,000 at the end of each of ten consecutive years.

#### Future Value of an Annuity for a Given Time

Let's assume that you have been the manager of a cooperative farm supply firm for twenty years and now expect to retire in five years. Your board of directors has just informed you that, in appreciation of your many years of service, they have decided to establish a special retirement fund for you. Each year, \$2,000 is to be invested in this special fund, which will receive an annual interest rate of ten percent. At the time of your retirement, five years hence, the fund will be given to you as a gift. What will be the total amount of money received at the time of your retirement? To answer this question, the following formula should be used:

$$R = \left[ \frac{(1+i)^n - 1}{i} \right]$$

where: A= future value

R= revenue (per conversion period)

i = interest rate (per conversion)

n = number of conversion periods

In our illustration, the formula becomes:

$$\begin{aligned} A &= \$2,000 \left[ \frac{(1.10)^5 - 1}{.10} \right] \\ &= \$2,000 \left( \frac{.6105}{.10} \right) \\ &= \$12,210 \end{aligned}$$

The result of your firm investing \$2,000 each of five consecutive years at ten percent interest, the revenues accumulating along with interest, will provide a retirement fund of \$12,210.

#### Rates and Relationships

In our illustrations, we used varying interest and discount rates. What effect does the selected rate have on the particular time-value relationship? In fact, the effect may be positive or negative, depending on the particular relationship to which we are referring. Table 1 illustrates the varying effects.

You will notice in Table 1 that increasing interest rates has a positive effect on the relationships concerned with future values, and increasing discount rates has a negative effect on the relationship's concerned with present values. This is, of course, completely logical as an increased rate simply speeds up the compounding or discounting process, the first increasing the future value and the latter decreasing the present value.

#### Time and Relationships

In our illustrations, we also chose to use varying time periods. What effect does the selected time period length have on the particular time-value relationship? Again, the effect varies. In determining the present

Table 1

The Effects of Rate Changes on Time-Value Relationships

<b>Time-Value Relationship</b>	<b>If Interest or (Discount) Rate:</b>	<b>Then:</b>	<b>Effect on Future or (Present) Value</b>
Future Value of Present Sum	<u>Increases</u>	Future Value Increases	Positive
Present Value of Future Sum	(Increases)	Present Value Decreases	(Negative)
Present Value of Annual Revenue	(Increases)	Present Value Decreases	(Negative)
Present Value of an Annuity	(Increases)	Present Value Decreases	(Negative)
Future Value of an Annuity	Increases	Future Value Increases	Positive

value of constant annual revenue, time period length has zero effect because the annual revenues are expected for an indefinite period of time. The other four time-value relationships, however, are affected differently by changes in the time period length selected. These varying effects are shown in Table 2. (See top of next page)

You will notice in Table 2 that increasing time period length has a positive effect on all relationships except for the present value of a future sum. Again, this is completely logical because as a given sum is to be received only after a longer period of time, one would expect its present value to decrease.

Diverse Periods

So far in our discussion, the conversion period in the illustrations has been identical with the revenue period, i.e., occurring once at the end of each year. For some agribusiness situations this may not be true, e.g., interest is often compounded

semiannually or monthly while revenues (or payments) occur annually, or revenues (or payments) occur monthly while interest is compounded quarterly.

When interest (or discounting) and revenues (or payments) are based on identical periods, our calculations are relatively simple, as shown. When they are based on diverse periods, however, the calculations become slightly more complex. Two adjustments must be made. First, the interest or discount rate must be stated in terms of a rate per conversion period, i.e., with ten percent interest compounded semiannually, this must be adjusted to five percent per conversion period. Second, the time period involved must be stated in terms of number of conversion periods, i.e., if interest is compounded semiannually for five years, the number of conversion periods will be ten.

The first three relationships discussed in this paper can be adjusted to diverse periods using

Table 2

The Effects of Time Period Length Changes on Time-Value Relationships

Time-Value Relationship	If time Period Length:	Then:	Effect on Future or (Present) Value
Future Value of Present Sum	<u>Increases</u>	Future Value Increases	Positive
Future Value of Future Sum	Increases	Present Value Decreases	(Negative)
Present Value of an Annuity	Increases	Present Value Increases	(Positive)
Future Value of an Annuity	Increases	Future Value Increases	Positive

the two-step conversion just described above. The two annuity relationships, however, require a third conversion before they become adaptable to diverse period. The annuity formulae used earlier included the term “n” which was defined as the number of conversion periods. Under diverse periods, the conversion period must be considered as the base period, i.e., the periodic revenue (or payment), the number of payments, and the interest or discount rate must be stated in terms of the conversion period.

Now we must add a new factor to our two annuity formulae, i.e., “e” is defined as the number of periodic revenues (or payments) in each conversion period. For example, if revenue is received monthly and discounting occurs semiannually, and then *e* is six. *R* was defined earlier as the revenue (payment) per conversion period. Under diverse periods, where revenue and conversion periods differ, we see that *R* now equals *e* multiplied by the dollar revenue (or payment).

When determining the present value of an annuity under diverse periods, we use the

same formula shown for identical periods except that we insert a correction factor which we shall denote as  $\frac{i}{je}$  as follows:

$$P = R \left( \frac{i}{je} \right) \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

where  $\frac{i}{je}$  is selected from a conversion factor table for the appropriate values of *e* and *i*. Using our earlier illustration on the purchase of a new two-ton truck, suppose we now change the percent annual discount rate to a semi-annual rate. The conversion period is now six months and our formula becomes:

$$\begin{aligned} P &= \$500(.9756) \left[ \frac{1 - (1.05)^{-20}}{.05} \right] \\ &= \$500(.9756) \left[ \frac{1 - .3767}{.05} \right] \\ &= \$6,080 \end{aligned}$$

where:

$P$  = present value

$R$  = revenue (per conversion period)  
= \$500 = 1/2 of \$1,000 received annually

$\frac{i}{je}$  = correction factor (table value for  $i, e$ )

$i$  = 5 percent discount rate (per conversion period)

$e$  = 1/2 = number of revenues (per conversion period)

$n$  = 20 = number conversion periods (during ten years)

If the discount rate had remained at ten percent annually, but the \$1,000 annual revenue were changed to \$500 received semiannually, then we see that  $R = \$1,000$ ,

$i = 10$ ,  $e = 2$ ,  $n = 10$ , and  $\frac{i}{je}$  would be

selected from the conversion table for the appropriate values of  $i$  and  $e$ .

If the future value of an annuity is to be determined with diverse periods, the correction factor would again be inserted into our earlier formula to form:

$$A = R \left( \frac{i}{je} \right) \left[ \frac{(1+i)^n - 1}{i} \right]$$


**Note:**

In working with annuities, we defined  $R$  as the revenue (or payment) per conversion period. In working with interest or discount rates, we referred to the future or present value of a sum. The illustrations used were described mostly as investments. Had we been referring to costs, rather than revenues (or payments), the calculations would not have changed, for a sum can be received as an income, or paid as a cost.

**Summary**

Five different relationships between time and the value of money have been described. Each relationship is designed to fit a different situation in so far as it applies to present or future value and discount or interest rates. Every agribusiness manager must make decisions concerning the investment of available capital. Before such decisions can be made intelligently, the manager must recognize, understand and apply numerous time-value relationships. This paper presents several time-value illustrations in an attempt to assist the manager in developing this recognition, understanding, and ability. The effects of changes in rates, time period lengths, and diverse periods on time-value relationships were also considered. Those relationships described were: a) future value of a present sum, b) present value of a future sum, c) present value of a constant annual revenue, d) present value of an annuity, and e) future value of an annuity.

For a more detailed discussion of the concepts noted in this paper, the reader is encouraged to write to the Superintendent of Documents, U. S. Government Printing Office, Washington, D.C. 20402 for a copy of Agricultural Handbook No. 349, "The Evaluation of Investment Opportunities" by Arthur J. Wabrath and W. L. Gibson, Jr., February 1968, U.S.D.A.



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