

STATISTICAL DECISION THEORY

an aid
to agribusiness
management

Cooperative Extension
College of Agriculture
& Home Economics
Washington State University
Pullman, Washington

STATISTICAL DECISION THEORY

an aid to agribusiness management

Ken D. Duft, Extension Marketing Economist Washington State University

Until a few years ago scientific aids to business management were used only when dealing with very specific problems such as inventory control. Recently, however, statistical decision theory has been developed and has been shown to have rather wide application to business management problems. It is this wide applicability that has made statistical decision theory so attractive to managers.

With due respect to its complexities, the basis of the managerial process is decision making. Statistical decision theory is merely a description - written in mathematical terms -of this aspect of the management process. As such, it is a valuable aid to scientific management.

The title of this workbook includes the word "statistical." It is used not to frighten off those less skilled in mathematics, but in recognition of the fact that today quantitative methods are essential shortcuts to the efficient expression of management methods and policies. Mathematical notation in no way makes the applicability of statistical decision theory less general. Instead, it simplifies application and understanding.

The purpose of this workbook is to show, via an illustrative example, how statistical decision theory can be applied to agribusiness management.

THE PROCEDURE

The most obvious place to begin our investigation of statistical decision theory is with some definitions. "Statistical" denotes reliance on a quantitative method. "Decision theory" describes a process, which results in the selection of the proper managerial action from among well-defined alternatives. By implication, "decision making" is the selection of the best alternative.

If statistical decision theory is to be applicable to the managerial process, it must adhere to each of the following elements of decision making:

- a) Definition of the problem.
- b) Establishment of the appropriate decision criteria.
- c) Accurate determination of the environmental situation.
- d) Description of all alternative managerial actions.
- e) Development of the decision process.

- f) Solution of the problem.
- g) Making the decision.

Each of the above elements will now be dealt with in more detail.

Definition of the Problem

All decision making problems can be characterized, in my opinion, by:

- a) The desire to attain an established goal.
- b) The availability of many alternative managerial actions.
- c) A particular environment, which exists with regard to the alternative actions, e.g., risk, certainty, uncertainty, conflict, ignorance.

The manager must analyze each problem in terms of each of the above characteristics. A solution is always more accessible when the manager has a more thorough understanding of the problem which he confronts.

For purposes of illustration, this discussion will be based on a simplified managerial problem. Note that the problem statement is directed toward the attainment of a particular goal.

Our example: John Fox is the manager of a relatively small agricultural supply firm. During the spring, his major supply item is a special type of chemical fertilizer. He sells it for \$14 a ton and pays \$8 per ton when he purchases it in bulk from a nearby manufacturer. Because of his small warehouse, Fox is unable to store any unsold fertilizer over the winter. He must, therefore, decide what is the most economic order quantity each spring. By returning to the past 10 years' records, Fox finds the following data on annual sales:¹

Figure 1

History of Sales

Sales (in tons)	Years of Given Sales
2,000	2
3,000	4
4,000	3
5,000	1

¹ Fox is not able to detect any noticeable sales trend over the years, nor is he able to relate annual sales with any environmental event.

Analyzing this problem in terms of the three characteristics set forth above, we determine:

- a) The goal--stated as a question--is how many tons of fertilizer should John Fox purchase this spring?
- b) The set of alternative managerial actions corresponds with the various amounts of fertilizer Fox could purchase, i.e. 2,000-5,000 tons.
- c) Upon consideration of the problem in light of the environment, it is decided that decision making will occur within a risk environment (based on the fact that past records have provided a probability distribution of annual sales).

Criteria Establishment

For our purposes, let us define a "decision criterion" as an indicator or index that would serve as an appropriate means of measuring attainment of the goal. In our example, we must ask what measure depicts the proper purchase quantity? The best possible criterion might be any of the following:

- a) Fox sells his complete supply of fertilizer each spring.
- b) Fox never turns down a customer for lack of fertilizer supplies.
- c) Fox maximizes short-run profit.
- d) Fox maximizes long-run profit.

We now notice that selection of a decision criterion involves making a subjective judgment on the part of the decision maker. By their very nature, subjective judgments are short-term in nature and, therefore, tend to vary with the problem composition. Some of the more common decision criteria are:

- a) Maximum absolute gain.
- b) Maximum expected gain.
- c) Minimum expected loss.
- d) Minimum absolute loss.

Maximum Absolute Gain is the decision criterion generally selected by the confirmed optimist (and gambler). He is the manager who operates on the "go for broke" philosophy and chooses the action with the greatest absolute gross profit², regardless of the risks associated with it. This criterion considers only the magnitude of the gross profits associated with each alternative and dictates the selection of the action with the largest absolute gross profit -regardless of the probability of attaining that profit. If Fox were a confirmed optimist and selected the Maximum Absolute Gain criterion, his resulting decision would be to buy 5,000

² Gross profit is equal to sales price less purchase price, e.g. in our problem, gross profit - \$6 = \$14 - \$8.

tons of fertilizer (Figure 2). If, in fact, fertilizer demands that year was 5,000 tons, he would receive the maximum absolute gross profit of \$30,000 (5,000 tons x \$6 per ton gross profit). Fox would make this decision despite his knowledge of the fact that there is only a 10 per cent chance that his annual fertilizer sales (demand) will reach 5,000 tons, i.e., only in one year of the past ten have sales reached this level.

Figure 2

Maximum Absolute Gross Gain

Potential Demand	Gross Profit Per Ton	Total Absolute Gross Gain
2,000	\$6	\$12,000.
3,000	\$6	\$18,000.
4,000	\$6	\$24,000.
5,000	\$6	\$30,000

Maximum Expected Gain, as a decision criterion, considers not only the absolute size of the potential gross profit but also the probability of attainment³. In our example, if Fox had selected the Maximum Expected Gain criterion, he would have preferred the purchase of 3,000 or 4,000 tons of fertilizer, each yielding an expected gain of \$7,200 (Figure 3).

³ For example, if there is a 20% probability of attaining a \$100 gross gain, then the expected gross gain of that event is \$20, i.e. \$100 x .20 = \$20.

Figure 3

Maximum Expected Gross Gain

(1) Purchase and Annual Demand	(2) Probability of Demand Occurring	(3) Total Absolute Gross Gain	(4)=(2)x(3) Expected Gross Gain
2,000	.20	\$12,000.	\$2,400.
3,000	.40	\$18,000.	\$7,200.*
4,000	.30	\$24,000.	7,200.
5,000	<u>.10</u>	\$30,000.	\$3,000.
	1.00		

* \$18,000. x .40 = \$7,200.

Minimum Expected Loss, as might be anticipated from the discussion above, involves the selection of the action with the smallest expected loss. In our example, because of the lack of proper storage facilities, unsold fertilizer deteriorates over the winter months and must be disposed of. A portion of Fox's total loss is, therefore, attributable to the cost of unsold fertilizer. The remaining part of the total loss is the "opportunity cost." This is both the foregone profit on unsold fertilizer and the unrealized profit on fertilizer sales, which would have occurred, had stock been available. As Figure 4 shows, this decision criterion involves the consideration of all possible losses and their related probabilities of occurrence⁴. According to the Minimum Expected Loss decision criterion, Fox would choose to purchase 3,000 tons of fertilizer.

⁴ For example, if 3,000 tons are stocked our data show that there is a 40% probability that demand (annual sales) will be the same as this, a 20% chance that demand will be less than this, and a 40% chance that demand will be more than this. Cost of unsold fertilizer equals the probability that demand will be less than purchase x quantity of excess x purchase' cost of excess, e.g. (for stock of 3,000 tons) \$1,600 = .20 x 1,000 x \$8. Overstock opportunity cost equals the probability that demand will be less than purchase x quantity of excess x forgone profit on excess, e.g. (for stock of 3,000 tons) = \$1,200 = .20 x 1,000 x \$6. Understock opportunity cost equals the probability that demand will be more than purchase x quantity of excess demand x forgone profit on excess demand, e.g. (for stock of 3,000 tons) \$3,000 = .30 x 1,000 x \$6 + .10 x 2,000 x \$6. Total expected loss equals cost of unsold fertilizer + overstock opportunity cost + understock opportunity cost.

Figure 4

Minimum Expected Loss

Purchase or Annual Demand	Probability			Expected			Total Expected Cost
	Demand Same as Purchase	Demand Less Than Purchase	Demand More Than Purchase	Cost of Unsold Fert.	Overstock Opport. Cost	Understock Opport. Cost	
2,000	.20	0	.80	0	0	\$7,800. ≠	\$ 7,800.
3,000	.40	.20	.40	\$ 1,600.	\$ 1,200.	\$3,000.	<u>\$ 5,800.</u>
4,000	.30	.60	.10	\$ 6,400.	\$ 4,800.+	\$ 600.	\$11,000.
5,000	.10	.90	0	\$13,600*	\$10,200.	0	\$23,800.

$$\begin{aligned}
 & * .20 \times 3,000 \times \$8 + .40 \times 2,000 \times \$8 + .30 \times 1,000 \times \$8 - \$13,600 \\
 & + .20 \times 2,000 \times \$6 + .40 \times 1,000 \times \$6 - \$4,800 \\
 & \neq .40 \times 1,000 \times \$6 + .30 \times 2,000 \times \$6 + .10 \times 3,000 \times \$6 \bullet \$7,800
 \end{aligned}$$

Minimum Absolute Loss, on the other hand, considers only the absolute magnitude of the possible losses and selects the smallest. This criterion will minimize loss if the worst possible event (a demand of only 2,000 tons in our example) occurs⁵. The manager choosing this decision criterion is often characterized as the "risk averter." All his decisions are based on a "play it safe" policy. Because the measure is absolute, it gives no consideration to the probabilities of occurrence of the losses. In other words, the smallest loss is chosen simply because it represents the smallest loss, and no attempt is made to calculate the probability of occurrence. As one might expect, in our example (Figure 5), given the worst possible occurrence, the purchase of 2,000 tons of fertilizer produces the minimum absolute loss of \$18,000. Had Fox chosen this decision criterion, he would have decided to purchase 2,000 tons.

⁵ In our example, the worst possible annual sales (2,000 tons) is expected to occur. All costs are computed as in Figure 4 except that they are based on an expected demand of 2,000 tons and no probabilities are involved in the calculations. The manager chooses that purchase quantity which minimizes total absolute loss.

Figure 5

Minimum Absolute Loss

Purchase or Annual Demand	Cost of Unsold Fertilizer	Worst Possible Overstock Opportunity Cost	Worst Possible Understock Opportunity Cost	Total Absolute Loss
2,000	0	0	\$18,000. ≠	\$18,000.
3,000	\$ 8,000.*	\$ 6,000.+	\$12,000.	\$26,000.
4,000	\$16,000.	\$12,000.	\$ 6,000.	\$34,000.
5,000	\$24,000.	\$18,000.	0	\$42,000.

- * 1,000 x \$8 = \$8,000
- + 1,000 x \$6 = \$6,000
- ≠ 3,000 x \$6 = \$18,000

Reviewing the previous discussion, one sees that depending on what decision criterion Fox chose, the managerial action selected varied among four alternatives:

<u>Decision Criterion Chosen</u>	<u>Scientifically Selected Managerial Action</u>
Maximum Absolute Gain	5,000 tons
Maximum Expected Gain	3,000 or 4,000 tons
Minimum Expected Loss	3,000 tons
Minimum Absolute Loss	2,000 tons

In fact, what has been shown is that four managers, each faced with the same goal, the same alternative managerial actions, and the same risk environment, could "scientifically" reach completely different managerial decisions. We have described, quantitatively, the process by which managers, faced with a common dilemma, disagree on the "best" solution. This disagreement depends on each manager's selection of an appropriate decision criterion.

Maximum Expected Net Value: Through the development of statistical decision theory, a fifth decision criterion may now be considered. Maximum Expected Net Value is defined as "the expected value of the profits minus the expected losses." This criterion considers all possible elements of the problem and is thus the most comprehensive measure. Furthermore, because it encompasses several of the previously described criteria (Figure 6), it is intuitively more attractive to most managers. The expected net value of each managerial action (j , where $j = 1, 2, \dots, 4$) is computed as follows:

$$ENV_j = EP_j - EL_j^6$$

Figure 6

Maximum Expected Net Value

Alternative Managerial Actions (tons)	Expected Profits (Epj) ⁷ (dollars)	Expected Losses (Elj) ⁸ (dollars)	ENVj ⁹ (dollars)
1 Pur. 2,000	2,400.	7,800.	-5,400.
2 Pur. 3,000	7,200.	5,800.	1,400.
3 Pur. 4,000	7,200.	11,800.	-4,600.
4 Pur. 5,000	3,000.	23,800.	-20,800

Throughout this discussion of decision criteria we have considered their measurement only in terms of dollars. Unfortunately, this is not always possible as certain value judgments cannot easily be stated in dollars. An example of such, is our criterion that stated that Fox might purchase fertilizer in such a volume as to insure that he never turned down a customer for lack of supplies. For non-monetary criteria such as this, or when dealing with a business manager who exhibits a non-linear utility function for money, economists are forced to suggest a set of decision criteria based on "utils" rather than dollars¹⁰. This general area is referred to as "utility theory" and will not be considered in this workbook.

Environmental Determination

If we are to define decision making as the process of selecting one alternative managerial action from among several, then we must also identify and describe the specific decision

⁶ Which reads, "Expected Net Value of managerial action j equals the expected profit of action j less the expected loss of action j ."

⁷ Derived from Maximum Gross Gain, Figure 3.

⁸ Derived from Minimum Expected Loss, Figure 4.

⁹ $ENV_j = (EP_j) - (EL_j)$

¹⁰ In fact, dollars are weighted by human desires before being taken into account.

making environment within which the manager finds himself. Probably the most typical environments of the business manager are:

- a) Certainty - an environment where the set of alternative managerial actions is known, and also the outcome of each action is known (with certainty).
- b) Risk - an environment where the set of alternative managerial actions is known, but where the outcome of each action can be estimated only in terms of a probability distribution.
- c) Uncertainty - an environment where the set of alternative managerial actions is known, but absolutely no information is available regarding the possible outcomes of each action.
- d) Conflict - an environment where the set of alternative managerial actions is known, but where the outcome of each action is dependent upon the reaction of equally knowledgeable opponents (competitors).
- e) Ignorance - an environment where the set of alternative managerial actions is unknown.

The list of decision making environments is in order of desirability. Naturally, all managers would prefer to operate under conditions of certainty. However, only rarely is a manager faced with such a pleasant environment. Decision making under risk is more commonplace. It is characterized by the availability of historical data from which probability distributions for the outcomes of alternative managerial actions can be constructed¹¹.

When the environment is one of uncertainty, the distinguishing characteristic is that historical data are not available, Often this lack is due to the fact that the process, itself, is of a non-repetitive nature. Or it may simply be due to the fact that no one has bothered to collect the data. A second distinguishing characteristic is that the manager generally has, or can obtain, an intuitive knowledge of the outcome probabilities. If this is not possible, the decision making environment would become one of ignorance and a scientifically derived management solution would be impossible.

Bayesian Statistics: The admission of intuitive (or subjective) judgments¹² into the decision making process has instigated a long and sometimes heated argument among statisticians. Those promoting the use of subjective probabilities are now referred to as "Bayesians." An essay published in 1763, by Bayes, an English Presbyterian minister, offered a statistical theorem on what is called "an inverse probability of an event." Its major use was for merging subjective probabilities with subsequently obtained sampling information. In its classical form, Bayes' Theorem reads:

¹¹ John G. Kemeny, Arthur Schleifer, Jr., J. Laurie Snill, and Gerald L. Thompson, *Finite Mathematics With Business Applications*, Prentice-Hall, Inc., Englewood, pp. 221-228.

¹² As opposed to objective or empirical probabilities

$$P(X / A) = \frac{[P(X) \cdot P(A / X)]}{P(A)} \text{ }^{13}$$

Bayes' Theorem provides a formal analytical description of a manager in the process of adjusting his opinion in the light of additional data.

The non-Bayesian statistician objects to this admission of subjective probabilities into statistical processes. He claims that if historical data are not available, the statistician cannot (and should not) attempt to describe the decision making procedure.

In all fairness, it should be noted that if subjective probabilities are not allowed to be considered, it becomes impossible for the manager to make rational decisions within an environment of uncertainty¹⁴. Over the years, "decision theory" seems to have become associated with decision making under an environment of uncertainty. However, this is generally not the case as the major part of statistical decision theory is relevant to a risk environment. In our farm supply example, John Fox is confronted with risk.

Description of Alternative Managerial Actions

The importance of this step should be obvious. It alerts the manager to his entire set of alternative actions. The importance of having a list as comprehensive as possible cannot be overemphasized. If, for some reason, the most appropriate managerial action is excluded from the list, even the most rigorous decision may not produce an effective solution.

The difficulty of enumerating every possible managerial action will, of course, vary with the complexity of the problem. In our example, the managerial actions available to John Fox are particularly simple to determine:

Managerial Action	Purchase Volume
1	2,000 tons
2	3,000 tons
3	4,000 tons
4	5,000 tons

Description of Outcomes

Once the manager has selected a decision criterion and identified the universe of alternative managerial actions, his next step, as a decision maker, is to evaluate these alternative actions. This evaluation is best accomplished by means of a mathematical process specifically

¹³ Which reads, "The probability of X occurring, given that A has occurred, equals the probability of X occurring multiplied by the probability of A occurring, given that X has occurred, all divided by the probability of A occurring."

¹⁴ This position also denies that experienced managers have the ability of developing a general "feeling" for their job.

designed to express the relationship between the environment and the alternative managerial actions in terms of the selected decision criterion. Since each problem is unique, no generalizations can be made as to an appropriate mathematical process. In our example, John Fox is concerned with the determination of the most efficient economic purchase quantity as measured by Maximum Expected Net Value. Fox's appropriate decision process (in mathematical notation) for this problem is:

$$\text{Maximize } E (P_j - L_j)$$

Where: P_j = the gross profit of the j th quantity of fertilizer purchases, i.e. the sales price of fertilizer sold less the cost of fertilizer purchases.

L_j = the opportunity costs and costs of unsold fertilizer associated with each j th purchase quantity.

E = the expected value of:

This process may not be totally clear at the moment, but we will return to it later. The important factor to note at the present is that the process was developed by the manager from his knowledge of the problem.

Unfortunately, statistics, alone, do not remove the need for the manager to thoroughly understand the problem. At best, the use of statistical aids serves to make consideration of a complex problem more precise.

It is also well to note that there may exist more than one appropriate decision process for each problem. The decision process merely conceptualizes the relationship between alternative managerial actions and the environment. It is quite possible to examine this relationship from various points of view. Therefore, the development and selection of the decision process is an expression of a value judgment as much as the selection of the decision criterion. However, regardless of the decision process selected, the manager must have a thorough knowledge of the problem in order to effectively apply statistical decision theory.

Problem Solution

In our example, the decision process determined was:

$$\text{Maximize } E (P_j - L_j)$$

The P subscript j , refers to the various profits in the process one for each purchase quantity. Therefore, P_1 is the profit associated with the purchase of 2,000 tons of fertilizer; P_2 is the profit from purchasing 3,000 tons; etc. L_j are losses, interpreted in a similar manner. The E represents "expected value."

The first step in problem solution is to compute P_j or gross profit for each purchase quantity. Because this gross profit is dependent on annual fertilizer demand (sales), it is sometimes referred to as "conditional" gross profit. When values of P_j are inserted into a

two-dimensional table (or matrix) of the form shown below, they are referred to as a "pay-off matrix" (Figure 7).

Figure 7

Pay-Off Matrix Showing Conditional Gross Profit

(P_j)

	Purchase Quantity (tons)			
Demand (tons)	2,000	3,000	4,000	5,000
2,000	\$12,000	\$ 4,000	\$- 4,000	\$-12,000+
3,000	12,000	18,000*	10,000	2,000
4,000	12,000	18,000	24,000	16,000
5,000	12,000	18,000	24,000	30,000

* Data on or below the main diagonal are computed by multiplying the sales price less purchase price per ton by the purchase quantity, i.e. $(\$14 - 8) \times 2,000 - 3,000 = \$18,000$. If demand exceeds purchase quantity, no account is taken here of understock opportunity costs.

+ Data above and to the right of the main diagonal are computed by multiplying the sales price less purchase price per ton by the demand quantity less the cost of unsold fertilizer as purchase quantity exceeds demand, i.e., $[(\$14 - 8) \times 2,000] - (3,000 \times \$8) = -\$12,000$.

As shown above, gross profits and cost of unsold fertilizer have been considered in constructing the pay-off matrix. Our decision process (i.e. maximum expected net value) requires that we also consider opportunity losses associated with both unsold fertilizer or unfilled demand. The opportunity losses associated with each purchase quantity also depend on demand and will, therefore, be referred to as conditional opportunity losses. The appropriate conditional opportunity loss matrix is shown below (Figure 8):

Figure 8

Matrix Showing Conditional Opportunity Loss

(L_j)

	Purchase Quantity (tons)			
Demand (tons)	2,000	3,000	4,000	5,000
2,000	0	\$6,000	\$12,000+	\$18,000
3,000	\$6,000*	0	6,000	12,000
4,000	12,000	6,000	0	6,000
5,000	18,000	12,000	6,000	0

* Data below the main diagonal refer to unfulfilled demand opportunity cost, i.e. the foregone profit per ton multiplied by the amount of excess demand (\$6 x 1,000 = \$6,000).

+ Data above and to the right of the main diagonal refer to overstock opportunity cost, i.e. the foregone profit per ton multiplied by the amount of overstock (\$6 x 2,000 = \$12,000).

The Maximum Expected Net Value decision process is based upon the concept of ($P_j - L_j$). So far we have computed only the conditional values and have not yet merged them to form conditional net values. Simple subtraction is used to derive the following matrix (Figure 9):

Figure 9

Matrix Showing Conditional Value ($P_j - L_j$)

	Purchase Quantity (tons)			
Demand (tons)	2,000	3,000	4,000	5,000
2,000	\$12,000*	\$ -2,000	\$-16,000	\$-30,000
3,000	6,000	18,000	4,000	-10,000
4,000	0	12,000	24,000	10,000
5,000	-6,000	6,000	18,000	30,000

* All elements refer to the respective P_j less the respects, L_j , i.e., \$12,000 - 0 = \$12,000 .

The final step in this decision process is to convert the conditional (absolute) values into expected values by multiplying through by the respective probability of each level of demand for fertilizer. It was previously determined in our example that historical probabilities would be used. These were:

<u>Demand</u>	<u>Probability of Occurrence</u>
2,000	.20
3,000	.40
4,000	.30
5,000	.10

If we multiply the $(P_j - L_j)$ matrix by the probability distribution we obtain the final solution of the form $E(P_j - L_j)$ shown below (Figure 10):

Figure 10

Solution Matrix Showing Expected Value of $(P_j - L_j)$

	Purchase Quantity (tons)			
Demand (tons)	2,000	3,000	4,000	5,000
2,000	\$2,400*	\$400	\$-3,200	\$-6,000
3,000	2,400	7,200	1,600	-4,000
4,000	0	3,600	7,200	3,000
5,000	-600	600	1,800	3,000
$E(P_1 - L_1)$	\$4,200	\$11,800	\$7,400	\$-4,000
$E(P_2 - L_2)$				
$E(P_3 - L_3)$				
$E(P_4 - L_4)$				

* All elements of each column equal those in Figure 9 multiplied by the respective probability of that event occurring, i.e., $\$12,000 \times .20 = \$2,400$; $\$6,000 \times .40 = \$2,400$; $0 \times .30 = 0$; $\$-6,000 \times .10 = \-600 . The sum of these weighted outcomes equals \$4,200.

This matrix shows that the expected value of $(P_j - L_j)$, as defined, is maximized by purchasing 3,000 tons of fertilizer annually. However, before we advise John Fox of the proper decision, let us take note of the appropriate characteristics of this solution:

- a) The problem situation was characterized as a “risk” environment, and empirical probabilities were used. Just how valid were the data? Do they still apply?
- b) The decision criterion selection was based on a value judgment of long-run profit needs, short-run cash needs, firm objectives, etc.
- c) The decision process was merely a conceptualization of the appropriate relationship between environment and managerial actions. Is such a relationship valid?
- d) For the final decision to be optimum, the list of alternative managerial actions must be all inclusive. Was it?

If the manager can live with each of the above, then one can safely say that statistical decision theory proved to be of vital importance in determining the "best" answer.

CONCLUSION

This workbook has shown how statistical decision theory can be applied to the solution of agribusiness management problems. The major points are:

- a) Statistical decision theory is applicable to decision making problems within an environment of risk or uncertainty.
- b) The decision theory process leads the manager to a "best" solution only if many complex value judgments have been properly made. These value judgments must be made in light of a thorough understanding of the problem.
- c) Statistical decision theory complements managerial experience and knowledge of business practices; it does not and cannot replace the basic function of a manager decision making.

REFERENCES

- Aigner, D. J., Principles of Statistical Decision Making, The Macmillan Company, New York, 1968.
- Bierman, Harold, Jr. and others, Quantitative Analysis for Business Decisions, (Rev. Edition), Richard D. Irwin, Inc., Homewood, Illinois, 1965.
- Bross, Irwin F., Design for Decision, The Macmillan Company; New York, 1963.
- Fellner, William, Probability and Profit, Richard D. Irwin. Inc., Homewood, Illinois, 1965.
- Freund, John E. and Irwin Miller, "In Defense of Frequencies", The American Statistician, April 1955, pp. 15-17, 27.
- Hadley, G., Introduction to Probability and Statistical Decision Theory, Holden Day, Inc., San Francisco, 1967.
- Hartley, H. O., "In Dr. Bayes' Consulting Room", The American Statistician, February, 1963, pp. 22-24.
- Harowitz, Ira, An Introduction to Quantitative Business Analysis, McGraw-Hill Book Company, New York, 1965, pp. 81-101.
- Luce, R. Duncan and Howard Raiffa, Games and Decisions, John Wiley and Sons, Inc., New York, 1957.
- Schlaifer, Robert, Introduction to Statistics for Business Decisions, McGraw-Hill Book Company, Inc., New York, 1961.
- Schlaifer, Robert, Probability and Statistics for Business Decisions, McGraw-Hill Book Company, Inc., New York, 1959.
- Weiss, Lionel, Statistical Decision Theory, McGraw-Hill Book Company, Inc., New York, 1961.

SAMPLE EXERCISE

Maize Processing Co.

Situation: Mr. Jones is the manager of the Mize Processing Company located in Whitetail, Washington. Mr. Jones’ organization processes and packs sweet corn for national distribution under the Good Taste label. All purchases of sweet corn are on contract with local growers. Tonnage purchases of sweet corn are contracted at \$200/ton. Processing and packing occurs only when a retail order is received and costs for this function have been running at \$50/ton. After processing and packing, Good Taste sweet corn sells for \$300/ton-equivlant to retail distributors. The processed sales history of Good Taste shows a variance from a low of 6,000 tons to a high of 9,000 during the past ten years, as shown below:

Figure 1

Good Taste Sales Record

<u>Year</u>	<u>Total Tons Sold to Retailer</u>	<u>Year</u>	<u>Total Tons Sold to Retailer</u>
1967	9,000	1962	6,000
1966	8,000	1961	7,000
1965	8,000	1960	8,000
1964	7,000	1959	9,000
1963	8,000	1958	7,000

Problem: If retail orders for Good Taste are not received, the sweet corn received under contract is not processed and must be disposed of because of a lack of any way to store it. If disposal occurs, it means a complete loss of the value of the product, e.g. \$200/ton. No pattern or trend exists on past years’ sales to retailers. At the beginning of each year, Mr. Jones must decide what volume of sweet corn the Maize Processing Co. will agree to purchase under contract. After the contracts are signed, no other source of supply exists and Mr. Jones must hope that the supply volume contracted will coincide, as closely as possible, with that year’s demand for the processed product.

Consider that you are Mr. Jones and, using statistical decision theory, decide on a contract volume for 1968. All relevant costs (including opportunity costs) are to be considered in your decision. Your decision should be based on a selection of a contract volume which will produce a Maximum Expected Net Value.

Answer to Sample Exercise

Contract Price of Sweet Corn	\$200/ton
Processing and Packing Cost	\$ 50/ton
Sale Price to Distributors	\$300/ton

<u>Tons Sold</u>	<u>Times Occurred</u>	<u>Probability of Occurring</u>
6,000	1	.1
7,000	3	.3
8,000	4	.4
9,000	<u>2</u>	<u>.2</u>
	10 yrs	1.0

Pay-Off Matrix

Possible Demand (tons)	Possible Contract Volumes (ton)			
	6,000	7,000	8,000	9,000
6,000	$[300-(200+50)] \times 6,000$ = \$300,000	$[300-(200+50)] \times 6,000$ - $(1,000 \times 200)$ = \$100,000	$[300-(200+50)] \times 6,000$ - $(2,000 \times 200)$ = -\$100,000	$[300-(200+50)] \times 6,000$ - $(3,000 \times 200)$ = -\$300,000
7,000	\$300,000	$[300-(200+50)] \times 7,000$ = \$350,000	$[300-(200+50)] \times 7,000$ - $(1,000 \times 200)$ = \$150,000	$[300-(200+50)] \times 7,000$ - $(2,000 \times 200)$ = -\$50,000
8,000	\$300,000	= \$350,000	$[300-(200+50)] \times 8,000$ = \$400,000	$[300-(200+50)] \times 8,000$ - $(1,000 \times 200)$ = \$200,000
9,000	\$300,000	= \$350,000	\$400,000	$[300-(200+50)] \times 9,000$ = \$450,000

Opportunity Loss Matrix

Possible Contract Volumes (tons)

Possible Demand (tons)	6,000	7,000	8,000	9,000
6,000	0	50 x 1,000 = \$50,000	50 x 2,000 = \$100,000	50 x 3,000 = \$150,000
7,000	1,000 x 50 = \$50,000	0	50 x 1,000 = \$50,000	50 x 2,000 = \$100,000
8,000	2,000 x 50 = \$100,000	1,000 x 50 = \$50,000	0	50 x 1,000 = \$50,000
9,000	3,000 x 50 = \$150,000	2,000 x 50 = \$100,000	1,000 x 50 = \$50,000	0

$(P_j - L_j)$ Matrix

Possible Contracts (tons)

Possible Demand (tons)	6,000	7,000	8,000	9,000
6,000	$300,000 - 0$ = \$3,000,000	$100,000 - 50,000$ = \$50,000	$-100,000 - 100,000$ = -\$200,000	$-300,000 - 150,000$ = -\$450,000
7,000	$300,000 - 50,000$ = \$250,000	$350,000 - 0$ = \$350,000	$150,000 - 50,000$ = \$100,000	$-50,000 - 100,000$ = - \$150,000
8,000	$300,000 - 100,000$ = \$200,000	$350,000 - 50,000$ = \$300,000	$400,000 - 0$ = \$400,000	$200,000 - 50,000$ = \$150,000
9,000	$300,000 - 150,000$ = \$150,000	$350,000 - 100,000$ = \$250,000	$400,000 - 50,000$ = \$350,000	$450,000 - 0$ = \$450,000

Expected Net Value Matrix

Possible Contract Volumes (tons)

Possible Demand (tons)	6,000	7,000	8,000	9,000
6,000	300,000 x .1 = \$30,000	\$50,000 x .1 = \$5,000	-200,000 x .1 = -\$20,000	-450,000 x .1 = - \$45,000
7,000	250,000 x .3 = \$75,000	350,000 x .3 = \$105,000	100,000 x .3 = \$30,000	-150,000 x .3 = -\$45,000
8,000	200,000 x .4 = \$80,000	300,000 x .4 = \$120,000	400,000 x .4 = \$160,000	150,000 x .4 = \$60,000
9,000	150,000 x .2 = \$30,000	250,000 x .2 = \$50,000	350,000 x .2 = \$70,000	450,000 x .2 = \$90,000
$\sum (P_1 - L_1)$	\$215,000			
$\sum (P_2 - L_2)$		\$280,000		
$\sum (P_3 - L_3)$			\$240,000	
$\sum (P_4 - L_4)$				\$60,000

Solution: In order to maximize Expected Net Value, Mr. Jones should decide to contract for the purchase of 7,000 tons of sweet corn during 1968.